# FLUIDIZATION OF A GRANULAR BED (THEORY)

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On the basis of a simple physical model, it is shown that four basic mechanisms are possible for the conversion of a granular bed into the fluidized state in apparatuses of constant cross section. Conditions for the practical realization of these mechanisms are discussed as functions of bed parameters.

Despite the extensive dissemination of fluidization technology in industry and the large number of empirical studies of the fluidized state under most diverse conditions (for example, see [1-5]), many fundamental phenomena and processes characteristic of fluidized systems have not yet received a satisfactory or completely unambiguous explanation. An example is the problem of initial fluidization when the flow of a fluidizing agent passing through a granular bed reaches some critical value.

The various phenomenological patterns in the conversion of a granular bed into the fluidized state are qualitatively well known and are described in numerous journals and monographs. However, there are extremely limited and frequently contradictory ideas about the causes leading to the realization of one actual pattern or another and about the influence of various factors on the initiation of the fluidized state. For example, to explain the maximum in the dependence of the pressure drop in the bed on the flow rate of the fluidizing agent and the deviation from the curve for ideal fluidization, one takes into account flow energy loss in acceleration of bed particles accompanying rearrangement immediately before the onset of fluidization [1] or in overcoming adhesive forces between particles or between particles and the wall [2, 4], nonuniformity of bed packing and of the coefficient of hydraulic resistance for particles occupying different positions in the bed [3, 5], the existence of "conserved" horizontal stresses which ensure a thrust force on the wall even for a reduction of the apparent weight of a granule to zero [5], etc. It is, therefore, necessary to construct a clear physical model of initial fluidization which would make it possible to explain observed phenomena from some single point of view.

Previously, the problem of conversion of a granular bed to the fluidized state was considered as a problem about the limiting equilibrium of a bed under conditions where only compressive stresses could be present in it [6]. In such a model, the onset of fluidization is identified with the time when one of the principal stresses goes to zero at some point in the bed, i.e., potential conditions are created for local breakdown and disruption of the bed. In particular, it was concluded there was simultaneous fluidization over the entire volume of a granular bed in apparatuses with vertical walls. In fact, from the discussion in [7], a state of limiting equilibrium ordinarily does not occur for the beds in actual apparatuses; the limiting condition for frictional forces is only reached at the walls of the apparatus but not within its volume. In addition, because of the existence of wall friction which hinders motion of the granular material, the appearance of slip areas in a bed does not in itself denote a conversion to the fluidized state.

A more realistic model of the onset of fluidization was proposed in [8], where three important facts were taken into consideration for the first time: the difference between the limiting state and the actual stress state of a granular bed, the possible existence of stresses that do not vanish with complete compensation of gravitational force by hydraulic interaction flow forces, and the low friction of the granular material at the boundary walls. The last item, in particular, made it possible to give a qualitative explanation of the phenomenon of plug formation observed in the fluidization of sufficiently deep beds. The model proposed below is in many ways similar to the model in [8].

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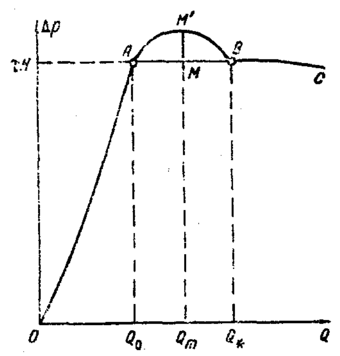


Fig. 1

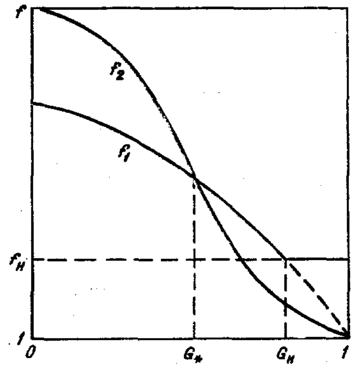


Fig. 2

Fig. 1. Fluidization curve in the absence of cohesive forces. The values $Q_0$ and $Q_*$ determine the range of flow rates in which gradual fluidization occurs, and the value $Q_m$ corresponds to the maximum pressure drop in the bed.

Fig. 2. Dependence of $f_1$ and $f_2$ on the parameter $G_3$. The curve for $f_1$ has a sharp bend at the point $(G_H, f_H)$. The curves cross at $G_3 = G_*$.

We characterize a granular bed by means of three parameters. First of all, we consider one-dimensional compression of a bed bounded in the lateral direction by smooth walls and we introduce the proportionality factor $\varkappa$ between the resultant transverse stresses and the applied longitudinal stress, which is assumed uniform [7, 8]. This quantity is a characteristic of the structure of the bed which appears as the result of its irreversible plastic deformation during establishment of a static equilibrium state of the bed in the apparatus. Therefore, $\varkappa$ depends essentially on the previous history of the granular bed, e.g., the method used for filling the apparatus with particles, the various dynamic effects, etc., and can be greater than one in a number of cases [8]. Therefore, the quantity $\varkappa$ should be considered as a characteristic of the initial state of a granular bed before fluidization.

If one now supposes that the applied compressive stress is reduced to zero, the transverse stresses generally do not vanish and the structure of the granular bed is essentially unchanged. In other words, transverse compression of bed granules is maintained together with a certain fraction of the corresponding elastic deformations which appeared earlier during the time of plastic deformation. In this sense, the latter deformations are "irreversible." The concept of the invariability of granular-bed structure up until the onset of fluidization and its essential rearrangement is discussed in [5] and actually follows from the experimental results of A. K. Bondarev for the electrical resistance of a filling of compressed steel balls as a function of the flow rate of an air stream [5]. According to those experimental results, the concept mentioned is indeed valid even for beds of smooth particles with so regular a geometric shape. The same conclusions can be reached on the basis of experiments with electrically conducting particles of irregular shape [9, 10].

The hypothesis concerning total independence of hydraulic pressure for the transverse stresses can be over-idealized, of course. In actuality, hydraulic forces lead to deformation of a bed as a kind of elastic medium characterized by its effective Poisson coefficient $\nu_e$ and the corresponding value

$$\varkappa_e = \nu_e(1 - \nu_e)^{-1}, \tag{1}$$

depending on particle shape, type of packing, and elastic constants. However, the structure of a bed is unchanged in the first approximation under such a deformation and the initial stresses are described as before by the coefficient $\varkappa$. A detailed discussion of such an "elastic filtration" mode is contained in [8].

Finally, we introduce the critical cohesive stress $\sigma_c$, which characterizes the interactions between the bed particles and is such that the disruption of the bed along some area becomes possible only if the normal stress on it corresponds to tension and is equal to or greater than $\sigma_c$. The physical reason for the appearance of cohesion can be adhesive forces, which are especially important in finely dispersed beds [11], electrostatic and magnetic attractive forces [12, 13], and also sticking of the particles because of their hygroscopic nature and the formation of liquid menisci in the contact areas [14, 15]. For simplicity, we neglect adhesion of particles to the walls of the apparatus, which in many cases is considerably weaker than the adhesion between particles.

The stress state of a granular bed in plane and cylindrical apparatus has been investigated [7] under the assumption that the limiting relation for frictional forces is only reached at the walls. As a result, expressions were obtained for the normal and shear stresses in a bed which are considerably simplified if the depth of the bed satisfies the inequality

$$H \ll \alpha R \,|1 - |1 - 2(1 + k)\varkappa\alpha^2|^{1/2}|. \tag{2}$$

In this case. we have approximately from [7]

$$\sigma_x^0 = \varkappa\gamma\left(1 - \frac{x^2}{R^2}\right)z, \quad \sigma_z^0 = \gamma z, \quad \tau = \varkappa\gamma\frac{xz^2}{R^2} \tag{3}$$

(compressive normal stresses are considered positive).

The condition (2) is satisfied for beds that are not too deep if the product $\varkappa\alpha$ is small in comparison with one; then

$$H \ll \frac{R}{(1+k)\varkappa\alpha}. \tag{4}$$

Since our main interest is the construction of a physical model and an investigation of the qualitative features of the conversion of a bed into the fluidized state, it is reasonable to consider condition (2) or condition (4) satisfied, which makes it possible to use the expressions (3) and to simplify the computations considerably.

With an ascending flow of the fluidizing agent, an upwardly directed hydraulic force

$$F = F(Q), \quad dF/dQ > 0, \tag{5}$$

acts on the particles in a unit volume which depends additionally on the viscosity of the medium, the size of the particles, and the porosity of the bed. Specific expressions for the function (5) were studied in many papers (see the review in [3-5]); in particular, the semiempirical formula of Ergun [16], which was well confirmed by experiment, yields fairly good results.

Using the method of [7] and assuming condition (2) or (4) is satisfied, we obtain the following expressions for the stresses in a granular bed having an ascending flow present by replacing $\varkappa$ with $\varkappa_e \leq \varkappa$:

$$\begin{gathered} \sigma_x = (\varkappa\gamma - \varkappa_e F)(1 - x^2/R^2)z, \quad \sigma_z = (\gamma - F)z, \\ \tau = \pm(\varkappa\gamma - \varkappa_e F)xz^2/R^2, \end{gathered} \tag{6}$$

which is true to the same approximation as Eqs. (3) for the initial stresses. Note that the upper sign in Eq. (6) should be selected for $\gamma > F$ when the wall friction forces are directed upward and the lower sign for $\gamma < F$.

The condition for the potential disruption of the bed at a level z below the free surface obviously has the form

$$[F(Q) - \gamma]z \geqslant \sigma_c. \tag{7}$$

It is clear that this expression can be satisfied for some z only when $Q > Q_0$, where $Q_0$ is a root of the equation

$$F(Q_0) = \gamma, \tag{8}$$

which is the commonly introduced minimum rate of fluidization.

If there is no cohesion between the particles and the initial packing of the filling is sufficiently loose so that the transverse compression of the particles, which is maintained when the flow reaches the critical value $Q_0$, is practically nonexistent (i.e., $\varkappa_e = \varkappa$), all parts of the filling are fluidized simultaneously when $Q = Q_0$ in complete agreement with earlier conclusions [6, 8]. In fact, the condition (7) is satisfied in this case over the entire volume of the filling, i.e., at all levels z, and the frictional forces which might prevent actual separation of particles in the vertical direction go to zero when $Q = Q_0$. This is the first of the mechanisms for conversion of a granular bed to the fluidized state; it should be observed when $\sigma_c = 0$ and $\varkappa_e = \varkappa$. In this case, the fluidization curve exhibits an "ideal" behavior (see the curve OAMBC in Fig. 1).

Now let $\sigma_c = 0$ as before but let $\varkappa_e < \varkappa$, i.e., there is a transverse compression of the particles which is not compensated by the resultant hydraulic forces. It is easy to see that when $Q > Q_0$, the condition (7) for potential disruption is satisfied over the entire granular bed. However, frictional forces at the walls prevent separation of individual horizontal layers of particles. The condition for fluidization at a depth z is obtained from the equation of balance for gravity, hydraulic pressure, and wall friction acting on an elementary layer of thickness dz and has the form

$$[F(Q) - \gamma] R^2 = (1 + k)[\varkappa\gamma - \varkappa_e F(Q)] z^2. \tag{9}$$

This condition determines the position of the "fluidization front" $z_*(Q)$, which is displaced downward as Q increases:

$$z_*(Q) = R\left[\frac{1}{1+k}\,\frac{F(Q) - \gamma}{\varkappa\gamma - \varkappa_e F(Q)}\right]^{1/2}. \tag{10}$$

Thus, in this case, fluidization is achieved layerwise beginning at the upper boundary of the filling in the range $(Q_0, Q_*)$ of the flow Q, with $Q_*$ being given by the equation

$$F(Q_*) = \gamma\,\frac{R^2 + (1+k)\varkappa H^2}{R^2 + (1+k)\varkappa_e H^2} > \gamma, \tag{11}$$

which follows from Eq. (9) for $z = H$. It is clear that the quantity $z_*$ becomes equal to H when $Q = Q_*$. The hydraulic resistance of the granular filling per unit cross-sectional area when $Q_0 < Q < Q_*$ is

$$\Delta p = \gamma H + [F(Q) - \gamma][H - z_*(Q)] > \gamma H \tag{12}$$

and goes to $\gamma H$ (bulk density of the bed) when $Q = Q_0$ or $Q = Q_*$. It is easy to note that the quantity (12) has a maximum for some $Q = Q_m$, $Q_0 < Q_m < Q_*$. This is a second possible mechanism for fluidization, which is realized in the absence of cohesion between particles. The dependence of the pressure drop (12) in the filling on the flow rate Q then has the form of the curve OAM'BC in Fig. 1 with a characteristic maximum and differs from the curve for ideal fluidization. We point out that if one now reduces the quantity Q, the state of the bed will obviously correspond to a point which is shifted in Fig. 1 toward the origin along the curve for ideal fluidization. Thus the well-known hysteresis of fluidization has a natural explanation.

The simplest form of the expressions given above is realized when $F(Q) = \beta Q$, i.e., for a linear law of hydraulic resistance. In particular,

$$Q_* = Q_0\frac{R^2 + (1+k)\varkappa H^2}{R^2 + (1+k)\varkappa_e H^2}, \quad z_*(Q) = R\left(\frac{1}{1+k}\,\frac{Q - Q_0}{\varkappa Q_0 - \varkappa_e Q}\right)^{1/2},$$
$$\Delta p = \gamma H[1 + (Q/Q_0 - 1)(H - z_*(Q))], \quad Q_0 = \gamma/\beta. \tag{13}$$

We now consider the features of fluidization of a granular filling characterized by a nonzero critical stress $\sigma_c$. In this case, the condition (7) primarily begins to be satisfied in the lower layers of the filling adjacent to the distributional grid at $z = H$. This occurs beginning at a flow value $Q_H$, where $Q_H$ is a root of the equation

$$F(Q_H) = \gamma + \sigma_c/H, \quad Q_H > Q_0. \tag{14}$$

Similarly, disruption of the filling along the plane z = const becomes possible if $Q \geq Q_z$, where

$$F(Q_z) = \gamma + \sigma_c/z, \quad Q_z \geqslant Q_H. \tag{15}$$

Further, the volume forces acting on the upper portion of the filling with a depth z will exceed the corresponding wall frictional forces upon the achievement of a flow with a value $Q_z'$ determined from

$$[F(Q_z') - \gamma] R^2 = \frac{1}{3}(1 + k)[\varkappa\gamma - \varkappa_e F(Q_z')] z^2 \tag{16}$$

[Eq. (6) for $\tau$ when $x = R$ was used in the calculation of the total frictional force]. It is obvious that at the time when the flows $Q_z$ and $Q_z'$ defined by Eqs. (15) and (16) become equal, a "plug" of thickness $z_s$, which begins accelerated motion upwards, should be separated from the upper portion of the filling. Relations for the determination of $z_s$ and of the critical flow $Q_s = Q_z = Q_z'$ at which such separation occurs follow directly from Eqs. (15) and (16). We have

$$[F(Q_s) - \gamma]^3 R^2 = \frac{1}{3}(1+k)[\varkappa\gamma - \varkappa_e F(Q_s)]\sigma_c^2, \quad Q_s \geqslant Q_H,$$

$$z_s = \sigma_c [F(Q_s) - \gamma]^{-1}, \quad z_s \leqslant H \tag{17}$$

[it is necessary to use the smallest of the roots of the first equation in (17) which exceeds $Q_H$]. In the particular case $\varkappa_e \ll \varkappa$, we have from (17)

$$F(Q_s) = \gamma + \left(\frac{1+k}{3}\frac{\varkappa\gamma\sigma_c^2}{R^2}\right)^{1/3}, \quad z_s = \left(\frac{3}{1+k}\frac{\sigma_c R^2}{\varkappa\gamma}\right)^{1/3}. \tag{18}$$

Thus for granular beds with cohesion between particles, a third mechanism is possible for the conversion into a fluidized state — sequential separation of finite portions of the bed which preserve their structure, forming a system of plugs. The individual plugs are separated by regions filled with fluidizing agent through which particles fall that drop from the lower surface of a plug onto the upper surface of the following plug. A similar conclusion about the nature of plug formation was reached earlier by Cherepanov [8].

Under the assumption made about insignificant cohesion between particles and apparatus walls, particularly the distributional grid, condition (7) is satisfied at the boundary between the bed and grid for any flow rate $Q \geq Q_0$. In principle, therefore, separation of the entire bed from the grid is possible at the time when the total hydraulic force acting on all particles in the bed becomes equal to sum of gravitational and wall friction forces. The latter occurs when $Q = Q_S'$, where $Q_S'$ is determined from the equation obtained from Eq. (16) when $z = H$:

$$[F_1(Q_s') - \gamma] R^2 = \frac{1}{3}(1+k)[\varkappa\gamma - \varkappa_e F(Q_s')] H^2. \tag{19}$$

If the quantity $Q_S'$ calculated from Eq. (19) turns out to be less than $Q_S$ evaluated from Eq. (17) or Eq. (18), a fourth type of onset of fluidization is observed. In this case, the entire bed is first separated from the grid, and then settling of particles from its lower boundary begins with subsequent conversion to the fluidized state. Thus, fluidization begins at the lower boundary of the bed. In the opposite situation ($Q_S' > Q_S$), fluidization occurs by means of the third mechanism. We emphasize that the root of Eq. (17) for $Q_S$ has a physical meaning only if it is greater than the quantity $Q_H$ defined in Eq. (14). If this requirement is not satisfied, it is necessary to assume $Q_S = Q_H$.

As is evident from an analysis of Egs. (14), (17), and (19), realization of the third or fourth mechanisms where cohesive force is present depends on the specific values of the three independent dimensionless parameters

$$G_1 = \frac{\varkappa\sigma_c^2}{\gamma^2 R^2}, \quad G_2 = \frac{\varkappa H^2}{R^2}, \quad G_3 = \frac{\varkappa_e}{\varkappa}. \tag{20}$$

It is easy to see that plug formation will occur if $f_1 < f_2$, and separation of the entire bed if $f_1 > f_2$, with the quantity $f_1$ being defined as a root of

$$(f_1 - 1)^3 = \frac{1+k}{3} G_1 (1 - G_3 f_1), \tag{21}$$

if this equation has a root greater than

$$f_H = 1 + \frac{\sigma_c}{\gamma H} = 1 + \left(\frac{G_1}{G_2}\right)^{1/2}, \tag{22}$$

and $f_1 = f_H$ otherwise; the quantity $f_2$ is given by

$$f_2 = \frac{1 + \frac{1}{3}(1+k) G_2}{1 + \frac{1}{3}(1+k) G_2 G_3}. \tag{23}$$

The dependence of $f_1$ and $f_2$ on the parameter $G_3$ for any fixed values of $G_1$ and $G_2$ is shown qualitatively in Fig. 2. The crossing of these curves determines a critical value of $G_3$ which, of course, depends on $G_1$ and $G_2$; plug formation is observed for values of $G_3$ less than the critical value, and bed separation for values of $G_3$ greater than the critical value.

All the specified types of conversion of a granular bed into a fluidized state have been observed experimentally many times [1-5]. In this regard, we note that according to the model presented, the physical analysis made by Zabrodskii [3] is the most correct and adequately reflects the essentials. At the same time, certain statements concerning the physical interpretation of this process widely disseminated in the literature are incorrect. For example, the explanation of the maxima in actual fluidization curves as an effect of particle cohesion [4] is incorrect. In actuality, these maxima are explained by the need to. overcome the wall frictional forces which are maintained in the system up to the time the flow reaches that minimal fluidization rate without which complete fluidization of the bed becomes impossible. In particular, the frictional forces are more important the greater the depth of the granular bed (other conditions being equal). Equations (11) and (12) show that the peak pressure drop in the fluidization curves also grows as the bed depth increases and the very process of total conversion of the bed to a fluidized state is extended over a broader range of flow rates. This viewpoint is confirmed by the experiments of Benenatti with beds of magnesium oxide particles in which there was observed a strong dependence of $Q_*$ and of the value of $\Delta P$ when $Q = Q_m$ on the depth of the bed (these experiments are described in [5]). However, we note that for depths H where condition (2) or (4) ceases to be valid, one must expect a weakening of the specified dependence with subsequent increase in H.

We note, in conclusion, that neither the quantity $\sigma_c$ nor the parameter $\varkappa$ are ordinarily measured in experiments with fluidized beds; this undoubtedly has a negative effect on the informational content of such experiments. Therefore, the organization of special experiments is obviously required for more refined judgement concerning the critical values of the parameters and also for checking the expressions obtained for critical flow rates, size of plug formations, etc.

## NOTATION

F, hydraulic force; $f_j$, $f_H$, parameters in Eqs. (21)-(23); k, parameter equal to zero and one, respectively, for flat and cylindrical beds; $\Delta p$, pressure drop; Q, flow rate (interstitial velocity); $Q_0$, $Q_*$, $Q_m$, critical values determined in Eqs. (8), (10), and (12), respectively; $Q_s$, incipient velocity of plug formation; $Q_H$, minimum value of $Q_s$; $Q_s'$, critical velocity for bed separation; R, halfwidth of flat, or radius of cylindrical, granular bed; x, z, horizontal and vertical coordinates, respectively; $\alpha$, wall friction coefficient; $\beta$, hydraulic force coefficient in Eq. (13); $\gamma$, effective specific weight of bed; $\varkappa$, proportionality constant between normal stresses for uniform compression of bed in its initial state; $\varkappa_e$, value of $\varkappa$ for elastic filtration; $\nu_e$, effective Poisson coefficient of bed; $\sigma_x$, $\sigma_z$, normal stresses; $\sigma_c$, critical cohesion stress; $\tau$, shear stress.

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